On Borrowing, Saving and the Technique of Gearing

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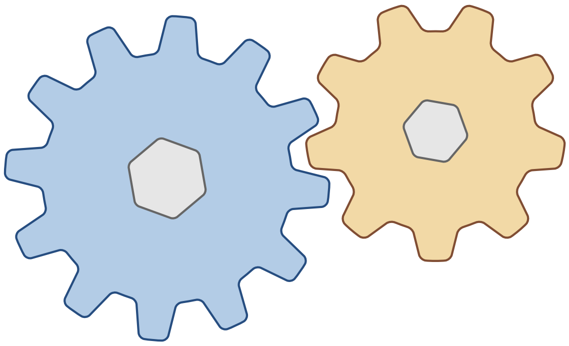

DOES IT MAKE FINANCIAL SENSE to borrow money when the borrowing itself comes at a cost? As always, the answer is, "It depends." For example, if a custodian of net worth anticipates that a property price will increase at a rate much higher than a mortgage rate, then it might well make sense to borrow now to purchase the property, despite the incurred borrowing cost. In this way, the custodian will be applying the technique of gearing.

Herein, I wish to analyse exactly when to apply such gearing. However, to do so, I first need to examine the dynamics of borrowing and of saving.

Borrowing. Let P_n be the component at time t_n of the custodian's net worth which is derived from the application of borrowed money. P_n could be the net asset value of a property, say. Then obviously

$$P_n = H_n - B_n \tag{1}$$

where H_n is sellable value of the corresponding asset at time t_n, and B_n is the value of the borrowed money at time t_n which is thus still owed to the lender. If we know that the asset appreciates over time at a rate η, say, and if the mortgage rate over time is μ, then we may write for any time t_n

$$\begin{aligned} H_n &= (1+\eta)H_{n-1} \\ B_n &= (1+\mu)B_{n-1} - M \end{aligned} \tag{2}$$

where M is the mortgage payment at time t_n which is required by the custodian in "servicing the loan."

The equations in (2) contain simple geometric series. So extrapolating from time t_0, it is easy to obtain:

$$\begin{aligned} H_n &= (1+\eta)^n H_0 \\ B_n &= (1+\mu)^n B_0 - \frac{(1+\mu)^n - 1}{\mu} M \end{aligned} \tag{3}$$

Substituting (3) into (1), we obtain

$$\boxed{P_n = (1+\eta)^n H_0 - (1+\mu)^n B_0 + \frac{(1+\mu)^n - 1}{\mu} M} \tag{4}$$

Equation (4) is the net asset value of the property at time t_n after the initial purchase at t_0. It is expressed in terms of the initial purchase price H_0, the initial amount borrowed B_0, the regular mortgage payments M, the property appreciation rate η, and the mortgage rate μ.

Paying back borrowing. As an aside, it is interesting to note that the expression for B_n in (3) is exactly that used by lending institutions to prescribe the value for the regular mortgage payments M. The institution would require the loan to be repaid after a time period of N months, say. That is, at t_N we require that:

$$B_N = 0 \tag{5}$$

Substituting (5) into (3) with $n = N$ gives:

$$\boxed{M = \frac{\mu(1+\mu)^N}{(1+\mu)^N - 1} B_0} \tag{6}$$

Typical numerical examples of regular monthly mortgage payments are shown in the following table.

Borrowed amount, B_0	*Loan term in months, N*	*Monthly mortgage rate, μ*	*Derived monthly mortgage payment, M*
400000	15*12	0.13/12	5060
400000	15*12	0.15/12	5598
400000	15*12	0.17/12	6156
600000	15*12	0.13/12	7591
600000	15*12	0.15/12	8397
600000	15*12	0.17/12	9234
800000	15*12	0.13/12	10121
800000	15*12	0.15/12	11196
800000	15*12	0.17/12	12312
1000000	15*12	0.13/12	12652
1000000	15*12	0.15/12	13995
1000000	15*12	0.17/12	15390

Saving. Let S_n be the component of the custodian's net worth at time t_n derived from making regular timely deposits into a saving instrument. A typical saving instrument is a moneymarket fund. If we know that the instrument appreciates at a rate σ, say, then we may write for any time t_n

$$S_n = (1+\sigma)S_{n-1} + D \tag{7}$$

where D is the regular timely deposit into the saving instrument. Similar to (3), the solution to S_n in (7) extrapolated from time t_0 is

$$\boxed{S_n = (1+\sigma)^n S_0 + \frac{(1+\sigma)^n - 1}{\sigma} D} \tag{8}$$

Equation (8) is the value of savings at time t_n after commencing at t_0, and assuming that n regular payment deposits have been made, each valued at D.

Gearing. Would it benefit the custodian to borrow money to purchase an appreciating asset whilst paying the regular mortgage installments M? Or would the custodian benefit more by simply accumulating savings through the regular payment deposites D? A simple criterion prescribing borrowing in favour of saving is to insist that at some future time t_m the net asset value P_m of the property must exceed the custodian's savings S_m:

$$P_m > S_m \tag{9}$$

for some future time t_m chosen by the custodian. A reasonable choice for m is simply $m = 1$. This is because the asset appreciation rate η in (2), the mortgate rate μ in (2), and the saving rate σ in (7) are probably best known for small m. Therefore, substituting (4) and (8) into (9), a criterion favouring borrowing becomes

$$hH_0 - bB_0 + M > sS_0 + D \tag{10}$$

We now consider the extreme cases in which the custodian either borrows fully and saves nothing, or saves fully and borrows nothing. In so doing, we are able to set $H_0 = B_0$, $D = M$ and $S_0 = D$ in (10), to give

$$\boxed{M < M_{\text{max}} = \frac{\eta - \mu}{1 + \sigma} H_0} \tag{11}$$

This is the *gearing criterion*. The custodian benefits by borrowing instead of saving only *only when the monthly mortgage payments are less than a certain threshold value*, M_{max}—a value determined by the three rates and the initial amount borrowed. As expected, the gearing criterion shows that the threshold value decreases with increase in saving rate σ. As σ increases, so does the attractiveness of saving increase relative to borrowing. Also, the threshold value decreases with increase in borrowing rate μ, thereby also increasing the attractiveness of saving. Conversely, also as expected, the threshold value for M in (11) increases with increase in asset appreciation rate η. As η increases, so does the attactiveness of borrowing increase relative to saving. But the custodian must be cautious because (11) shows that it is the difference $\eta - \mu$ which is important, and not η on its own.

In the following table, the gearing criterion (11) was used to calculate upper bounds on the monthly mortgate M.

Initial borrowing for asset purchase, H_0	*Proportional monthly increase in:*[§] *Asset value,* η	*Borrowing,* μ	*Saving,* σ	*Borrow, do not save, only if mortgage payment* M *less than...*
Saving rate σ equals 10% p.a.				
1000000	0.10/12	0.15/12	0.10/12	-4132
1000000	0.15/12	0.15/12	0.10/12	0
1000000	0.20/12	0.15/12	0.10/12	4132
1000000	0.30/12	0.15/12	0.10/12	12396
Saving rate σ equals 15% p.a.				
1000000	0.10/12	0.15/12	0.15/12	-4115
1000000	0.15/12	0.15/12	0.15/12	0
1000000	0.20/12	0.15/12	0.15/12	4115
1000000	0.30/12	0.15/12	0.15/12	12345
Saving rate σ equals 20% p.a.				
1000000	0.10/12	0.15/12	0.20/12	-4098
1000000	0.15/12	0.15/12	0.20/12	0
1000000	0.20/12	0.15/12	0.20/12	4098
1000000	0.30/12	0.15/12	0.20/12	12295

[§]*Note: The borrowing rate μ is set to equal a Prime Lending Rate of 15% p.a.*

While the gearing criterion (11) places an upper bound on the monthly mortgage payments M in order for borrowing to make fiscal sense to the custodian, the lending institution has its own idea of what those mortgage payments must be. If the institution expects its loan to the custodian to be repaid after N months, say, then the institution would need to apply (6). Therefore, combining (6) with (11) and using $B_0 = H_0$ offers, after some simple algebra, a new gearing criterion:

$$\boxed{\eta > \eta_{\text{min}} = \mu \left(1 + \frac{1 + \sigma}{1 - (1 + \mu)^{-N}} \right)} \tag{12}$$

According to this new criterion, the custodian benefits by borrowing instead of saving only when the anticipated property appreciation rate η exceeds a certain minimum value, η_{min}—a value determined by the mortgate lending rate μ, the saving rate σ, and the term of the mortgate loan N. This gearing criterion (12) was used to compute the lower bound on the anticipated property appreciation rate η as a function of the mortgate rate μ. This is shown in Figure 1. For example, with a saving rate of $\sigma = 0.15/12$ per month and a mortgate loan term of $N = 15 \times 12$ months, Figure 1 shows that at a mortgate rate of $\mu = 0.1/12$ per month, the property would have to appreciate at an anticipated $\eta = 0.23/12$ per month for gearing to make fiscal sense.

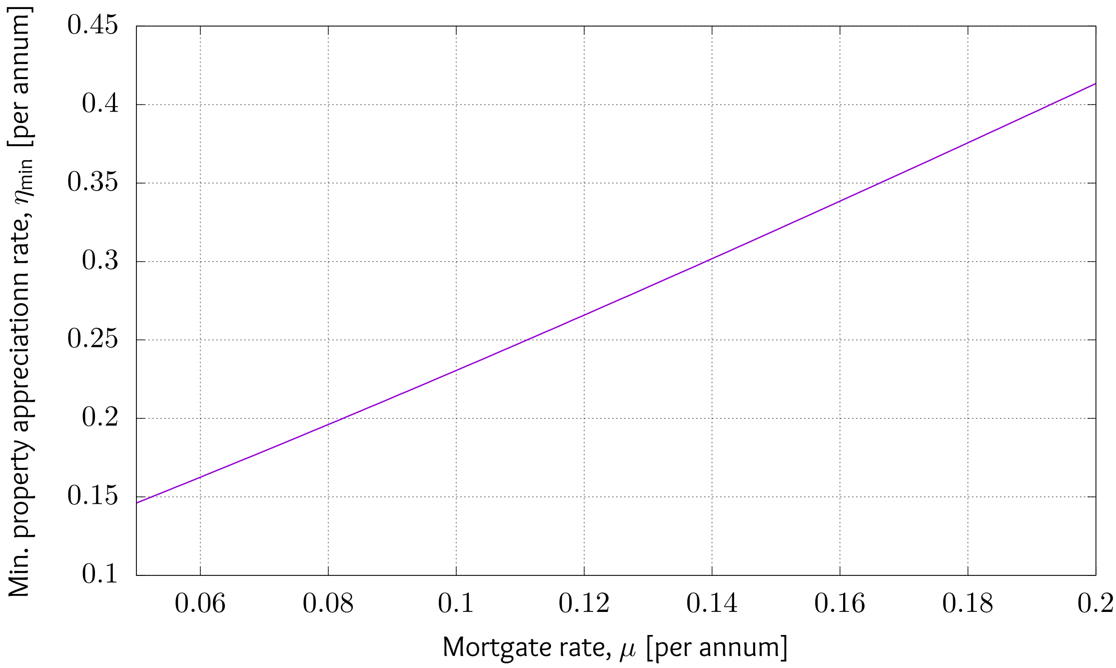

Figure 1: Plot of the minimum anticipated property appreciation rate η_{min} as a function of the mortgate lending rate μ. Equation (12) was used to compute $\eta_{\text{min}}(\mu)$ with the saving rate set to $\sigma = 0.15/12$ per month and the loan term set to $N = 15 \times 12$ months.

Of course, the all important quantities η_{min}, μ and σ are difficult to predict far into the future. But estimating them in simple repeated applications of the gearing criteria (11) or (12) will help give the custodian some sense of the inteplay between risk and reward.

In closing. The numerical examples above expose the sensitivity of gearing to the difference between the asset appreciation rate and the borrowing rate, $\eta - \mu$. They also expose its relative insensitivity to the saving rate, σ.

The above analysis, together with the numerical results, neglects any possible supplementary contributions to the custodian's net worth through ownership of the asset. Such contributions might be in the form of rent charged to a property tenant. Because such contributions may be signficant, this work must be extended to account for them.